

LINEAR ALGEBRA_A Date : April 26, 2024
Time : 13.45 – 15.45 hrs

First read these instructions carefully:

This test contains 9 exercises. The complete solutions of Exercises 2, 4, 5, 6, 7, and 9 must be accurately written down including calculations and argumentation and for the other exercises you are only required to fill in the final answers on the answer sheet at the end of this test. You must hand in the answer sheet.

The use of dictionaries or electronic devices is not allowed.

The grade is determined by dividing the total number of points by 4 and adding 1.

1. (2 pts) *Fill in your final answer to this exercise on the supplied answer sheet.*

For the system of equations with augmented matrix shown below, give the solution in parametric vector form.

$$\left(\begin{array}{ccccc|c} 1 & 2 & -1 & 0 & -2 & 2 \\ 0 & 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right)$$

2. *Use the supplied space and include clear argumentation and calculation.*

Let $A = \begin{pmatrix} 1 & 2 \\ 2 & 6 \end{pmatrix}$.

a) (2 pts.) Find A^{-1} , the inverse of A .

b) (2 pts.) By using A^{-1} , solve the system $A\mathbf{x} = \mathbf{b}$ where $\mathbf{b} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$, that is, find $\mathbf{x}$.

3. (2 pts) *Fill in your final answer to this exercise on the supplied answer sheet.*

If A , B , and C are invertible matrices of size $n \times n$ each, solve the equation $C^{-1}(A + X)B^{-1} = I_n$ for X .

4. (2pts) *Use the supplied space and include clear argumentation and calculation.*

Let $\mathbf{u} = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}$, $\mathbf{v} = \begin{pmatrix} 5 \\ -13 \\ -3 \end{pmatrix}$, and $\mathbf{b} = \begin{pmatrix} -3 \\ 8 \\ 1 \end{pmatrix}$. Then $\text{Span}\{\mathbf{u}, \mathbf{v}\}$ is a plane through the origin in $\mathbb{R}^3$. Is $\mathbf{b}$ in the mentioned plane? Motivate your answer.

5. Use the supplied space and include clear argumentation and calculation.

$$\text{Let } A = \begin{pmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & -1 & 1 \\ 0 & 1 & -1 & -1 \end{pmatrix}.$$

a) (3 pts) Determine Null A .

b) (2 pts) Find a basis for Null A .

c) (2 pts) Determine the coordinates of $\begin{pmatrix} 1 \\ -1 \\ 7 \end{pmatrix}$ with respect to the basis for Col A given by

$$\mathcal{B}_{\text{Col } A} = \left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} \right\}.$$

6. (4 pts.) Use the supplied space and include clear argumentation and calculation.

Let $B \in \mathbb{R}^{n \times n}$. Given $B^T \cdot B = I_n$, show that either $\det B = 1$ or $\det B = -1$.

7. Use the supplied space and include clear argumentation and calculation.

$$\text{For the given matrix } A = \begin{pmatrix} 1 & -2 \\ 1 & 3 \end{pmatrix}$$

a) (2 pts.) Find all eigenvalues of A .

b) (2 pts.) Find the corresponding eigenspaces of A .

8. (4 pts.) Fill in your final answer to this exercise on the supplied answer sheet. Each error results in a one-point deduction from the total score (4 pts).

Determine whether each of the following statements is true or false.

- i. If $\mathbf{v}$ is an eigenvector of A , then $\mathbf{v}$ is also an eigenvector of A^2 .
- ii. For an invertible matrix A , if λ is an eigenvalue of A , then λ is also an eigenvalue of A^{-1} .
- iii. For an invertible matrix A , if $\mathbf{v}$ is an eigenvector of A , then $\mathbf{v}$ is also an eigenvector of A^{-1} .
- iv. If $\mathbf{v}$ is an eigenvector of A , then $\mathbf{v}$ is also an eigenvector of A^T .
- v. If $\lambda \neq 0$ is an eigenvalue of AB (for square matrices A and B) then λ is also an eigenvalue of BA .

9. Use the supplied space and include clear argumentation and calculation.

Let G and F be linear transformations from $\mathbb{R}^2$ to $\mathbb{R}^2$. The linear transformation F rotates the vectors around the origin over an angle of $\frac{\pi}{2}$ radians. The representation matrix of $G \circ F$ is given by $\begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix}$.

a) (2 pts.) What is the representation matrix of F ?

b) (2 pts.) What is the representation matrix of G ?

c) (3 pts.) Is $G \circ F$ one-to-one? Is $G \circ F$ onto? Motivate your answers.

Do not remove the staple!

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Answer sheet Answer the questions legibly in the spaces provided. If you run out of room for an answer, continue on the back of the answer sheet by clearly referring to the question.

Student Name & Student Number: _____

Programme: _____

Do not write anything on this table.										
Question:	1	2	3	4	5	6	7	8	9	Σ
Max. Points:	2	2+2	2	2	3+2+2	4	2+2	4	2+2+3	36
Points Earned										

1. Provide only your final answer.

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2.a) Show all your work.

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2.b) Show all your work.

3. Provide only your final answer.

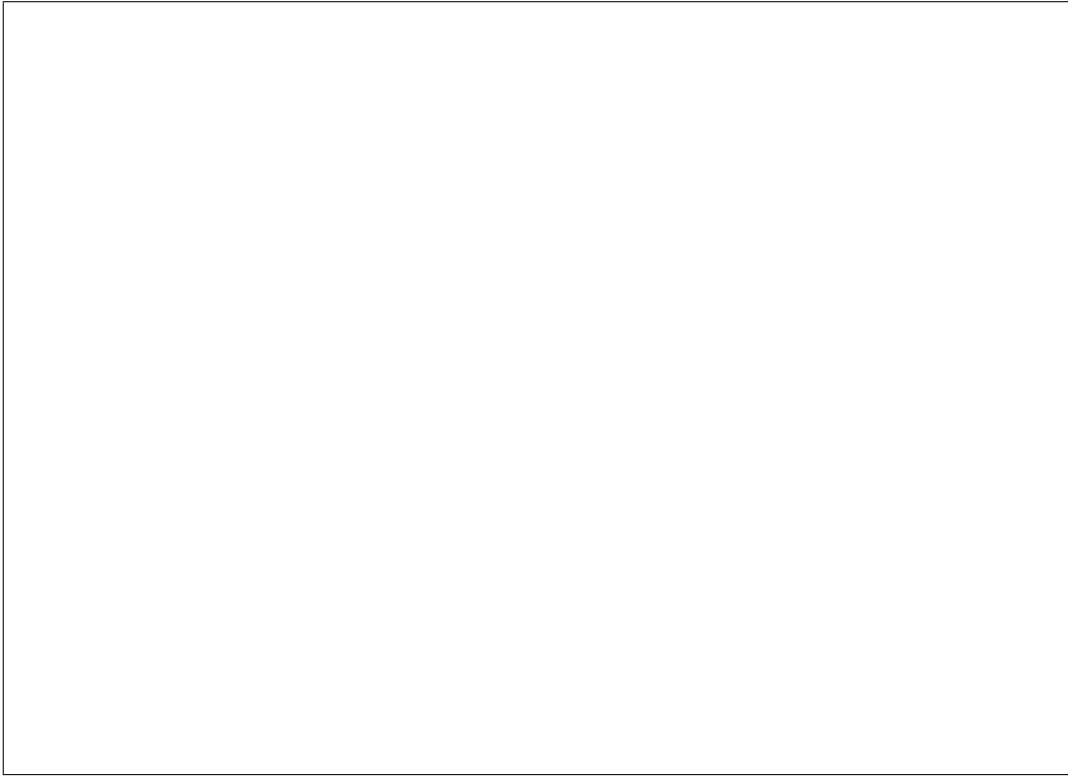
4. Show all your work.

5.a) Show all your work.

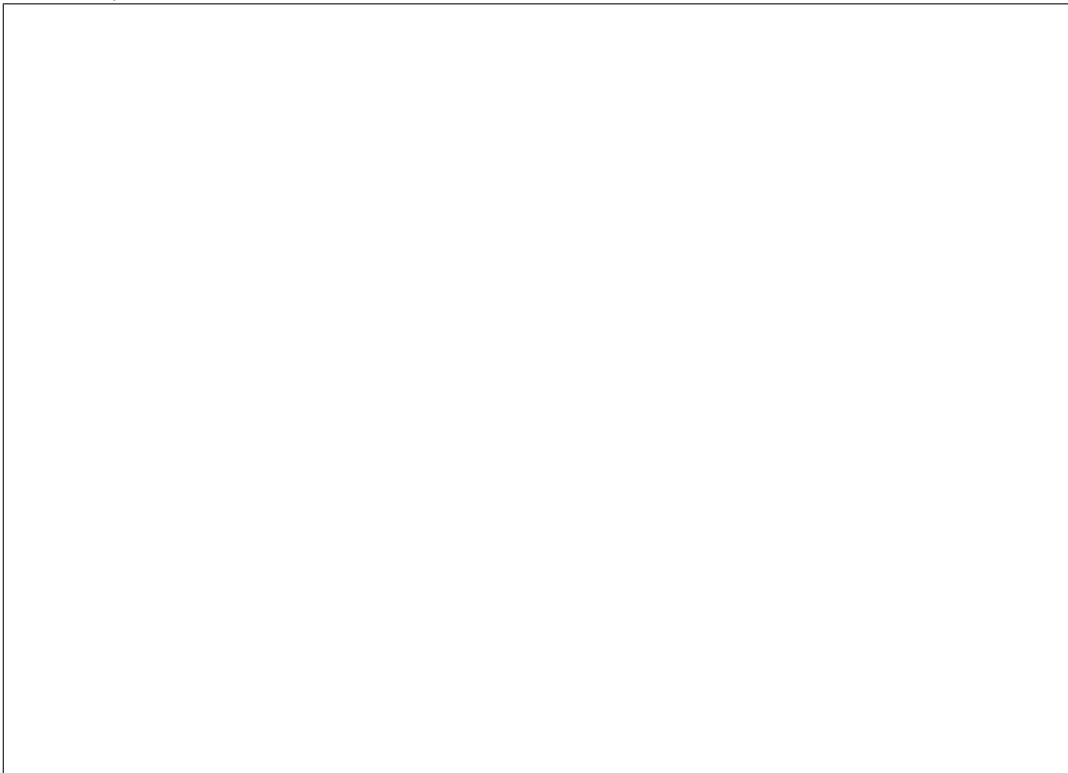
5.b) Show all your work.

5.c) Show all your work.

6. Show all your work.

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7.a) Show all your work.

A large, empty rectangular box with a thin black border, intended for the student to show their work for problem 7.a).

7.b) Show all your work.

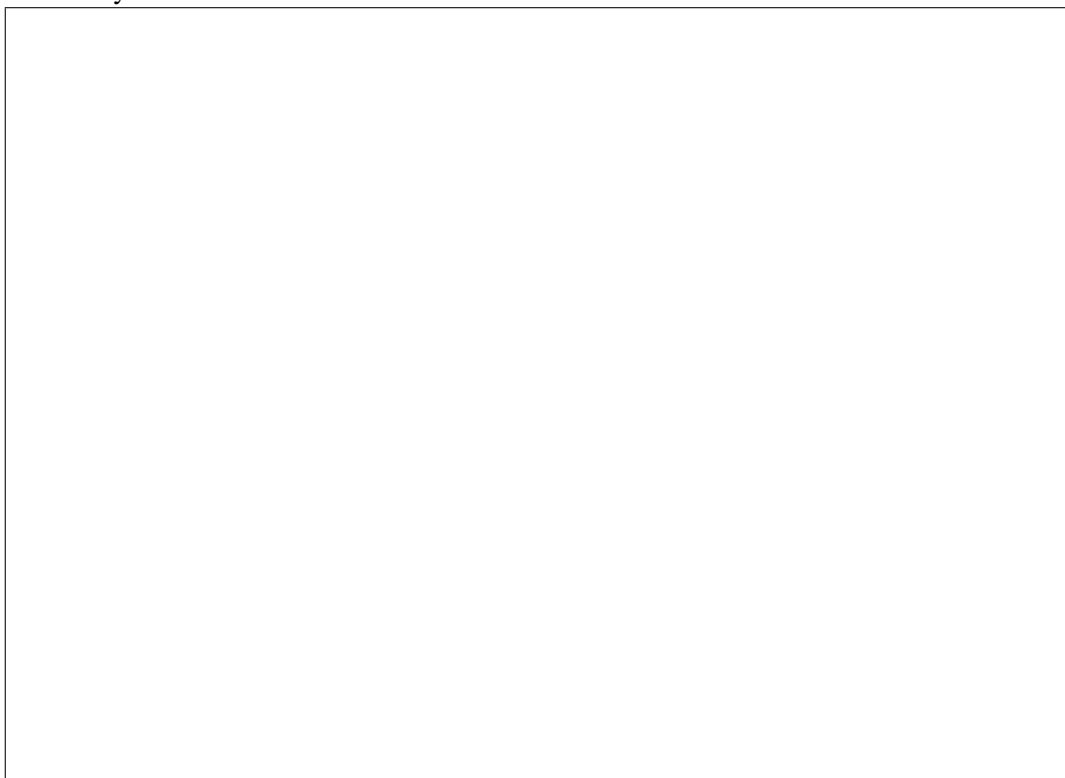
8. Use a check mark (✓) to choose the correct answer.

- | | | | | |
|------|--------------------------|------|--------------------------|-------|
| i. | ☐ | True | ☐ | False |
| ii. | ☐ | True | ☐ | False |
| iii. | ☐ | True | ☐ | False |
| iv. | ☐ | True | ☐ | False |

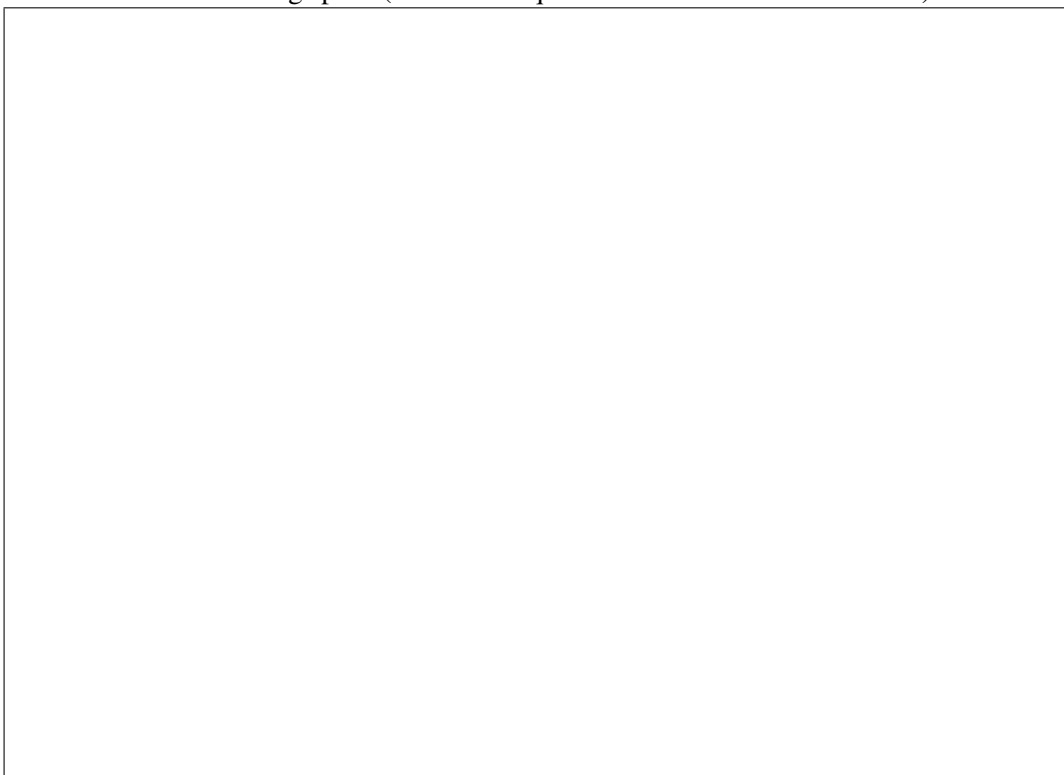
9.a) Show all your work.

9.b) Show all your work.

9.c) Show all your work.

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Extra writing space (Include the question number for each answer!)

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Extra writing space (Include the question number for each answer!)